good agreement with the literature values and the values of σ_r at 3.6986 GHz are close to the expected values calculated from their dc resistivity. Even in the case of the samples of wider range of resistivity used by Champlin *et al.*, the values calculated from the relations suggested in the present paper and those obtained from the complex transcendental equation, show an agreement within 1–2 percent for almost all the samples, although for the sample with resistivity 3 $\Omega \cdot$cm the agreement is slightly reduced. The reduced accuracy for very low resistivity samples ($\rho \sim 1\ \Omega \cdot$cm) is compensated by the ease with which σ_r and ϵ_r can be evaluated from (23) and (21).

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Short Papers

Permittivity Measurement of Modified Infinite Samples by a Directional Coupler and a Sliding Load

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***Abstract*—A cross coupler and waveguide sliding short technique for measuring the permittivity of an infinite sample is described in this paper. The experimental results obtained for commercially available cement, wheat flour, magnesium oxide, potassium bromide, glycerin, and water are given together with the estimated error. In view of the growing industrial use of microwaves, moisture dependent ϵ-values for cement and wheat flour are also reported.**

I. Introduction

A number of techniques for measuring the permittivity of dielectric materials at microwave frequencies have been reported [1] and a general review can be found in an excellent survey by Lynch [2]. These methods can be generally divided into three groups: transmission line methods, resonant methods, and perturbation methods [3]. The infinite sample method [4] is particularly suitable for routine measurements as a function of temperature. This is because of the convenience of temperature control and the simplicity with which the permittivity can be calculated from the experimental data. However, when the dielectric constant is large and/or the loss tangent is high, the uncertainty of measurements in conventional systems increases very rapidly. A method reported recently by Stuchly *et al.* [5] requires accurate measurement of the resonant frequency and the Q factor of the resonators for determining the permittivity of these materials.

The method presented in this paper calls for a four-port directional coupler and a precision sliding short. It has already been shown by the author [6] that the coupler and sliding short arrangement can be used for measurement in place of the slotted section. The dielectric constants of commercially available cement, wheat flour, magnesium oxide, potassium bromide, glycerin, and water are determined by this technique. Dependence of the permittivities of cement and wheat flour on moisture content is also studied by this method in X-band.

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II. Theoretical Relationships

The input impedance of a homogenous nonmagnetic dielectric filled semi-infinite rectangular waveguide carrying TE mode is [5]

$$Z=\left[1-(\lambda/\lambda_c)^2\right]^{1/2}/\left[\epsilon-(\lambda/\lambda_c)^2\right]^{1/2} \qquad (1)$$

where λ is the free space wavelength, λ_c is the cutoff wavelength

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for the mode of propagation in an empty guide, and $\epsilon = \epsilon' - j\epsilon''$ is the permittivity of the material. Thus from (1)

$$\epsilon' = (\lambda/\lambda_c)^2 + \frac{\left[1-(\lambda/\lambda_c)^2\right]\left[S^2 \sec^4 \beta d_1 - (1-S^2)^2 \tan^2 \beta d_1\right]}{(1+S^2\tan^2\beta d_1)^2} \quad (2)$$

$$\epsilon'' = \frac{\left[1-(\lambda/\lambda_c)^2\right]\left[2S(1-S^2)\sec^2\beta d_1 \tan\beta d_1\right]}{(1+S^2\tan^2\beta d_1)^2} \quad (3)$$

where S is the VSWR, d_1 is the location of the first minimum from the load, and β is the propagation constant in the feeding guide.

The uncertainty in ϵ' and ϵ'' resulting from errors in the VSWR measurements are calculated from (2) and (3) as

$$\frac{\delta\epsilon'}{\epsilon'} = \frac{\left[1-(\lambda/\lambda_c)^2\right]\left[A\delta S + B\delta(\beta d_1)\right]}{\epsilon'(1+S^2\tan^2\beta d_1)^2} \quad (4)$$

and

$$\frac{\delta\epsilon''}{\epsilon''} = C\delta S + D\delta(\beta d_1) \quad (5)$$

where

$$A = 2S\sec^4\beta d_1 + |4S(1-S^2)\tan^2\beta d_1| + 4S\tan^2\beta d_1(1+S^2\tan^2\beta d_1)\frac{\epsilon'-(\lambda/\lambda_c)^2}{1-(\lambda/\lambda_c)^2}$$

$$B = 4S^2\tan\beta d_1\sec^4\beta d_1 + 2(1-S^2)^2\tan\beta d_1\sec^2\beta d_1 + 4S^2(1+S^2\tan^2\beta d_1)\tan\beta d_1\sec^2\beta d_1\frac{\epsilon'-(\lambda/\lambda_c)^2}{1-(\lambda/\lambda_c)^2}$$

$$C = \frac{1}{S} + \left|\frac{2S}{1-S^2}\right| + \frac{4S\tan^2\beta d_1}{1+S^2\tan^2\beta d_1}$$

and

$$D = 2\tan\beta d_1 + 2\csc(2\beta d_1) + \frac{4S^2\tan\beta d_1\sec^2\beta d_1}{1+S^2\tan^2\beta d_1}.$$

III. Experimental Results

The experimental setup used for determining the dielectric constant of the materials was similar to that of the impedance measurement by cross coupler [6], in which the unknown load was a waveguide filled with a sample and the rear port of this guide was terminated by a matched load [4]. The tuners were used [7] to minimize the error in VSWR due to finite directivity of the coupler. The VSWR's were measured by adjusting the sliding short positions for maximum and minimum outputs of the detector. The well known double-minimum method was used for determining the VSWR whenever it exceeded 4. The location of the first minimum for each VSWR was determined after short circuiting the load port of the cross coupler. The dielectric constants of cement, wheat flour, magnesium oxide, potassium bromide, glycerin, and water determined by this method are shown in Table I along with the estimated errors, assuming a precision short of $\Gamma = -0.98 \pm 0.01$ with a least count of 0.01 mm. In estimating the errors in ϵ' and ϵ'', uncertainties in S and βd_1 were taken as

$$\delta S = \frac{0.03(S^2-1)}{1.97-0.03S}$$

and

$$\delta(\beta d_1) = 0.0028 \text{ rad}.$$

The experimentally determined moisture-dependent ϵ-values for cement and wheat flour are shown in Fig. 1.

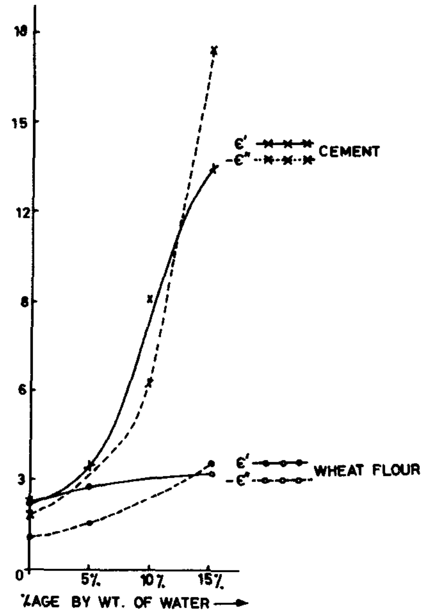

Fig. 1. Dielectric constant versus humidity for cement ($f = 9.375$ GHz) and wheat flour ($f = 9.389$ GHz) at room temperature.

TABLE I

Sample	Dielectric constant[1] ϵ'	$-\epsilon''$	$\delta\epsilon'/\epsilon'$	$\delta\epsilon''/\epsilon''$	Frequency, in gigahertz
Cement	2.23	1.77	0.167	0.179	9.375
Wheat flour	2.28	1.07	0.074	0.132	9.389
Magnesium oxide	3.24	1.58	0.098	0.164	9.375
Potassium Bromide	2.83	1.19	0.079	0.146	9.39
Glycerin	4.08	2.94	0.162	0.214	9.388
Water	59.74	31.11	0.503	0.717	9.8

[1]Measured at room temperature.

IV. Conclusion

The permittivities of cement, wheat flour, magnesium oxide, potassium bromide, glycerin, and water as found by this method, are shown in Table I, which are fairly in agreement with the available data [8]. The estimated errors show that this method is not suitable for materials having large dielectric constants and/or high loss tangents. The permittivity of water determined by this method is, by chance, in good agreement, though is has an uncertainty of more than 50 percent.

Dependence of ϵ' and ϵ'' of cement and wheat flour on the moisture content, is illustrated in Fig. 1. It shows a rapid change in the values of ϵ' and ϵ'' in case of cement in comparison to wheat flour. In all the measurements, the samples in powder form were hand pressed in waveguide. The dielectric constant will differ with change in pressure.

An application of the cross coupler and sliding short is demonstrated by the author in this paper. The dielectric constant can be measured by this technique even at high microwave power exposing the sample in the waveguide. However, uncertainty analysis shows that this method is not suitable when the dielectric constant and/or loss tangent is high.

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Impedance Transformation Equations for Exponential, Cosine-Squared, and Parabolic Tapered Transmission Lines

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***Abstract*—Closed-form equations that give the value of an arbitrary complex impedance transformed through a length of dissipationless, nonuniform transmission line with exponential, cosine-squared, and parabolic taper are presented. These equations are obtained by solving a second order nonlinear differential (Riccati) equation relating impedance, the nonuniform line impedance and the line length. The results presented should be useful in solving impedance matching problems.**

I. Introduction

Several articles [1]–[8] have dealt with the analysis and application of nonuniform transmission lines. However, most of these articles suggest the use of graphs or the Smith chart for impedance matching problems and cannot be readily used when the load impedance is complex instead of real. To this end we derive closed form impedance transformation equations, which can be used to calculate the value of an arbitrary complex impedance when transformed along the length of a dissipationless transmission line with exponential, cosine-squared, and parabolic taper.

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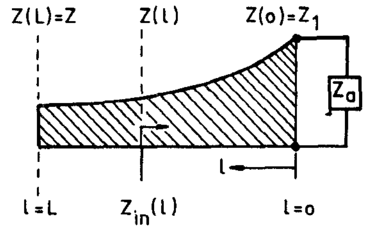

Fig. 1. Schematic representation of a generalized tapered transmission line.

First, a general second order nonlinear differential (Riccati) equation relating input impedance, the nonuniform line impedance and the line length is derived (Section II). This equation is then applied to exponential, cosine-squared, and parabolic tapered lines and solutions are given.

II. Derivation of Differential Equation for Input Impedance

In this section a second order nonlinear differential equation for input impedance of tapered transmission line is derived. For a uniform transmission line of characteristic impedance Z_0, a load impedance Z_a when transformed a distance l towards the generator is Z_{in} [9]

$$\frac{Z_{in}}{Z_0}=\frac{Z_a+jZ_0\tan\beta l}{Z_0+jZ_a\tan\beta l}. \tag{1}$$

Impedance of a tapered nonuniform transmission line is variable, it is a function of distance l, thus Z_0 becomes $Z(l)=Z$ (for brevity). To determine the expression for input impedance consider Fig. 1, and let Z_{in} be input impedance at l and $(Z_{in}+dZ_{in})$ at $(l+dl)$. If we assume the line to be uniform of "characteristic impedance" $Z(l)=Z$ over the incremental distance dl, then using (1)

$$\frac{Z_{in}+dZ_{in}}{Z}=\frac{Z_{in}+jZ\tan(\beta dl)}{Z+jZ_{in}\tan(\beta dl)}. \tag{2}$$

For small $\beta dl, \tan(\beta dl)=\beta dl$, and ignoring product of differential terms [10]

$$\frac{dZ_{in}}{dl}=\frac{-j\beta Z_{in}^2}{Z}+j\beta Z. \tag{3}$$

Equation (3) is the well-known Riccati equation, which can be transformed into homogeneous linear differential equation of second order and then solved [11]. Making the transformation

$$Z_{in}=\frac{Z}{j\beta}\left(\frac{1}{u}\frac{du}{dl}\right). \tag{4}$$

Then from (3) and (4)

$$\frac{d^2u}{dl^2}+\left[\frac{1}{Z}\frac{dZ}{dl}\right]\frac{du}{dl}+\beta^2u=0. \tag{5}$$

III. Exponentially Tapered Line

Along an exponential taper, $\ln Z(l)$ varies linearly with l

$$\ln Z=\ln Z_1+\frac{l}{L}\ln(Z_2/Z_1). \tag{6}$$

Differentiating with respect to l, and using it in (5)

$$\frac{d^2u}{dl^2}+\left\{\frac{1}{L}\ln\left(\frac{Z_2}{Z_1}\right)\right\}\frac{du}{dl}+\beta^2u=0. \tag{7}$$